**EECE 210 – Quiz 3**

**November 21, 2015**

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1. Simplify the circuit to a single capacitor in parallel with a single inductor, given that *C*1 = 2 μF, *C*2 = 3 μF, *L*1 = 20 mH, *L*2 = 30 mH.

**Solution:** *Ceq* = *C*1*C*2/(*C*1 + *C*2), *Leq* = *L*1*L*2/(*L*1 + *L*2).

**Version 1:** *C*1 = 2 μF, *C*2 = 3 μF, *L*1 = 20 mH, *L*2 = 30 mH; *Ceq* = 1.2 μF, *Leq* = 12 mH

**Version 2:** *C*1 = 20 mF, *C*2 = 30 mF, *L*1 = 2 μH, *L*2 = 3 μH; *Ceq* = 12 mF, *Leq* = 1.2 μH

**Version 3:** *C*1 = 10 μF, *C*2 = 15 μF, *L*1 = 40 mH, *L*2 = 60 mH; *Ceq* = 6 μF, *Leq* = 24 mH

**Version 4:** *C*1 = 40 mF, *C*2 = 60 mF, *L*1 = 10 μH, *L*2 = 15 μH; *Ceq* = 24 mF, *Leq* = 6 μH

**Version 5:** *C*1 = 4 μF, *C*2 = 6 μF, *L*1 = 1 mH, *L*2 = 1.5 mH; *Ceq* = 2.4 μF, *Leq* = 0.6 mH



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**B.** Given that *i*(*t*) = 2cos5*t* A. Express *v*(*t*) as a phasor in rectangular coordinates.

**Solution:** *ωL* = 5 Ω, -1/*ωC* = -1/(5×0.1) = -2 Ω. Hence **V** = 2(10 + *j*5 –*j*2) = 20 + *j*6 V.

**Version 1:** *i*(*t*) = 2cos5*t* A; **V** = 2(10 + *j*3) = 20 + *j*6 V

**Version 2:** *i*(*t*) = 3cos5*t* A; **V** = 3(10 + *j*3) = 30 + *j*9 V

**Version** **3:** *i*(*t*) = 4cos5*t* A; **V** = 4(10 + *j*3) = 40 + *j*12 V

**Version 4:** *i*(*t*) = 5cos5*t* A; **V** = 5(10 + *j*3) = 50 + *j*15 V

**Version 5:** *i*(*t*) = 6cos5*t* A; **V** = 6(10 + *j*3) = 60 + *j*18 V.

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**C.** Determine the average power dissipated in the 10 Ω resistor in the preceding problem.

**Solution:** *P* = (*Irms*)2*R* = (4/2)×10 = 20 W

**Version 1:** *i*(*t*) = 2cos5*t* A; *P* = (*Irms*)2*R* = (4/2)×10 = 20 W

**Version 2:** *i*(*t*) = 3cos5*t* A; *P* = (*Irms*)2*R* = (9/2)×10 = 45 W

**Version 3:** *i*(*t*) = 4cos5*t* A; *P* = (*Irms*)2*R* = (16/2)×10 = 80 W

**Version 4:** *i*(*t*) = 5cos5*t* A; *P* = (*Irms*)2*R* = (25/2)×10 = 125 W

**Version 5:** *i*(*t*) = 6cos5*t* A; *P* = (*Irms*)2*R* = (36/2)×10 = 180 W.

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**D.** Coil 1 of 100 turns and Coil 2 of 200 turns are wound on a high-permeability core. When coil 1 current is 2 A, with coil 2 open-circuited, the flux in the core is 3 mWb. Determine the mutual inductance of the coils.

**Solution:** The flux linking coil 2 is 600 mWb-T. The mutual inductance is 600/2 = 300 mH

**Version 1:** *I*1 = 2 A; *M* = 600/2 = 300 mH

**Version 2:** *I*1 = 3 A; *M* = 600/3 = 200 mH

**Version 3:** *I*1 = 4 A; *M* = 600/4 = 150 mH

**Version 4:** *I*1 = 5 A; *M* = 600/5 = 120 mH

**Version 5:** *I*1 = 6 A; *M* = 600/6 = 100 mH.

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1. *Rin* for the circuit shown is 2 Ω when *R* = 1 Ω. Determine *Rin* of the dual circuit.

**Solution:** *Gin* of the dual circuit in S is numerically

equal to *Rin* in Ω. It follows that *Rin* of the dual

circuit in Ω is the reciprocal of *Rin* of the given circuit.

**Version 1:** *R* = 1 Ω, *Rin* = 2 Ω; *Rin*(dual) = 0.5 Ω

**Version 2:** *R* = 2 Ω, *Rin* = 4 Ω; *Rin*(dual) = 0.25 Ω

**Version 3:** *R* = 3 Ω, *Rin* = 6 Ω; *Rin*(dual) = 0.167 Ω

**Version 4:** *R* = 4 Ω, *Rin* = 8 Ω; *Rin*(dual) = 0.125 Ω

**Version 5:** *R* = 5 Ω, *Rin* = 10 Ω; *Rin*(dual) = 0.1 Ω.

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1. The phasor (1 – *j*) is rotated 90° clockwise. Represent the rotated phasor in the time domain as the imaginary component of the complex sinusoidal function of time, assuming an angular frequency *ω* rad/s.

**Solution:** When rotated clockwise, the phasor becomes (1 – *j*)/*j* = -1 – *j* = . The complex sinusoidal time function is . The imaginary part is  .

**Version 1:** phasor (1 – *j*) → -sin*ωt* – cos*ωt*

**Version 2:** phasor 2(1 – *j*) → -2sin*ωt* – 2cos*ωt*

**Version 3:** phasor 3(1 – *j*) → -3sin*ωt* – 3cos*ωt*

**Version 4:** phasor 4(1 – *j*) → -4sin*ωt* – 4cos*ωt*

**Version 5:** phasor 5(1 – *j*) → -5sin*ωt* – 5cos*ωt*.

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1. Determine *Zin* if *Z* = (1 + *j*) Ω.

**Solution:** With a test source **VT** applied, **VT** = *Z***IZ** and **IT** = (1 – *j*)**IZ**.

Dividing these two equations,  Ω.

**Version 1:** *Z =* (1 + *j*) Ω; *Zin* = *j* Ω

**Version 2:** *Z =* 2(1 + *j*) Ω; *Zin* = *j*2 Ω

**Version 3:** *Z =* 3(1 + *j*) Ω; *Zin* = *j*3 Ω

**Version 4:** *Z =* 4(1 + *j*) Ω; *Zin* = *j*4 Ω

**Version 5:** *Z = 5*(1 + *j*) Ω; *Zin* = *j*5 Ω.

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1. Determine *iC*(*t*) given that *vSRC*(*t*) = 2sin(5*t* + 30°) V.

**Solution:** *ωL* = 5×4 = 20 Ω; -1/*ωC* = =1/(5×0.01) =

-20 Ω. It follows that the parallel combination is an open circuit, so that *vSRC* appears across this combination. Hence *iC*(*t*) = *CdvSRC*/*dt* = 0.05×2cos(5*t* + 30°) A.

**Version 1:** *vSRC*(*t*) = 2sin(5*t* + 30°) V; *iC*(*t*) = 0.05×2cos(5*t* + 30°) = 0.1cos(5*t* + 30°) A

**Version 2:** *vSRC*(*t*) = 4sin(5*t* + 30°) V; *iC*(*t*) = 0.05×4cos(5*t* + 30°) = 0.2cos(5*t* + 30°) A

**Version 3:** *vSRC*(*t*) = 6sin(5*t* + 30°) V; *iC*(*t*) = 0.05×6cos(5*t* + 30°) = 0.3cos(5*t* + 30°) A

**Version 4:** *vSRC*(*t*) = 8sin(5*t* + 30°) V; *iC*(*t*) = 0.05×8cos(5*t* + 30°) = 0.4cos(5*t* + 30°) A

**Version 5:** *vSRC*(*t*) = 10sin(5*t* + 30°) V; *iC*(*t*) = 0.05×10cos(5*t* + 30°) = 0.5cos(5*t* + 30°) A.

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1. Determine *M* so that no current flows in the 10 Ω resistor, given that *L* = 3 H and *ω* = 1 rad/s.

**Solution:** Using the T-equivalent circuit, the circuit in the frequency domain is as shown. For zero current through the middle branch, . Solving for *M*,  H.

**Version 1:** *L* = 3 H; *M* = 5/4 = 1.25 H

**Version 2:** *L* = 5 H; *M* = 7/6 = 1.167

**Version 3:** *L* = 7 H; *M* = 9/8 = 1.125

**Version 4:** *L* = 11 H; *M* = 13/12 = 1.083

**Version 5:** *L* = 21 H; *M* = 23/22 = 1.045.

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1. Determine *vO*(*t*) as a cosine function, given that *iSRC*1(*t*) = cos10*t* A and *iSRC*2(*t*) = sin10*t* A.

**Solution:**  3 H. Applying superposition, *vO*(*t*) = -*MdiSRC*1/*dt* + *L*2*diSRC*2/*dt =* 30*Im*sin10*t* + 40*Im*cos10*t* = 50(cos*θ*cos10*t* + sin*θ*sin10*t*) = 50*Im*cos(10*t* – 36.9°) V.

**Version 1:** *Im* = 1; *vO*(*t*) = 50cos(10*t* – 36.9°) V

**Version 2:** *Im* = 1.5; *vO*(*t*) = 75cos(10*t* – 36.9°) V

**Version 3:** *Im* = 2; *vO*(*t*) = 100cos(10*t* – 36.9°) V

**Version 4:** *Im* = 2.5; *vO*(*t*) = 125cos(10*t* – 36.9°) V

**Version 5:** *Im* = 3; *vO*(*t*) = 150cos(10*t* – 36.9°) V.

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1. Determine the power dissipated in the 4 Ω resistor by each independent source acting alone, given that *ISRC* = 0.25 A dc and *vSRC*(*t*) = 10cos(103*t* + 30°) V.

**Solution:** Under dc conditions, the circuit is as shown. *IX* = 0.25 A; 3*IX* = 0.75

A; the power dissipated is (0.75)2×4 =

2.25 W.

Under ac conditions, *ωL* = 103×10-3 = 1 Ω; -1/*ωC* =

-1/103×10-3 = -1 Ω; it follows that the inductor in series with the capacitor is equivalent to a short circuit, The circuit becomes as shown. From KVL around the mesh on the LHS, **VSRC** + 4**IX** – 3**IX**(-*j*) = 0, or **IX** = -**VSRC**/(4 + *j*3). It follows that |**IX**| = |**VSRC**|/5 = 2 A. The power dissipated in the 4 Ω resistor is  8 W.

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1. Derive TEC looking into terminals ‘ab’, given that V. Represent Thevenin’s voltage in the time domain and express *ZTh* in rectangular coordinates.

**Solution:** Using the T-equivalent circuit, (*L*1 – *M*) = 1 H and (*L*2 – *M*) = 0. From KVL, the voltage across the lower

*j*Ω inductor is (1 – *j*)**IO** and the downward current through

this inductor is (1 – *j*)**IO**/*j* = (-1 – *j*)**IO**. The current through

the upper *j*Ω inductor is (-2 – *j*)**IO** in the direction shown.

From KVL about the outer loop, not including the

current source, **VSRC** = *j*(-2 – *j*)**IO** + **IO** = 2(1 – *j*)**IO**. It

follows that **IO** = **VTh** = , which in the time domain is *vTH*(*t*) = 2cos(*t* + 90°) = -2sin*t* V.

 On short circuit, **IO** = 0, so that both dependent sources are set to zero, and the *j* Ω inductor in the middle is short circuited. It follows that **ISC** = **VSRC**/*j*. Hence, *ZTh* = Ω.

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1. Determine *kX* so that no power is delivered

or absorbed by *vSRC*2, given that *vSRC*1(*t*) = 10cos10*t* V, *vSRC*2(*t*) = 10sin10*t* V, and *k* = 0.5.

**Solution:** *ωL* = 10×1 = 10 Ω; -1/*ωC* =

-1/10×0.01 = -10 Ω; 10sin10*t* = 10cos(10*t* – 90°),

*jωM* = Ω. The circuit

in the frequency domain is as shown.  A. It follows that -*j*10 = 1(*j*10 – *j*10) – *j*5 + *j*5 – , which gives *kX* = 1.